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Abstract

We discuss integrated chance constraints in their role of short-term risk constraints in a strategic ALM model for Dutch pension funds. The problem is set up as a multistage recourse model, with special attention for modeling the guidelines proposed by the regulating authority for Dutch pension funds. The paper concludes with an outline of a special-purpose heuristic, which is used to approximately solve the resulting model which contains many binary decision variables.

Key words: modeling, ALM, integrated chance constraints, multistage mixed-integer recourse

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1. Introduction

The strategic Asset Liability Management (ALM) problem for pension funds is a *dynamic* decision problem under *uncertainty*. Management of assets involves decisions on the strategic portfolio mix, and the liabilities – consisting of (future) pension payments – depend on indexation policies. Because of the long time horizon over which the liabilities range, typically in the order of 30 years, the problem is inherently dynamic. Moreover, uncertainty plays a major role because e.g. asset investments yield unknown pay-offs; by way of the valuation of future liabilities at market value, this is also a source of uncertainty on the liability side.

The goal of the ALM process is to enable payment of current and future pensions. This should be done at minimal funding costs, consisting of contributions by active participants of the fund and the sponsor (e.g., the company backing the fund), and subject to laws and the rules specified by the regulation authority for pension funds. In addition, the outcome has to confirm to the long-term policy rules of the pension fund.

Thus, in general terms the ALM problem is to select *decisions* on allocation of the assets, the contributions, indexation of future payments (relative to e.g. wage inflation), etcetera, which are *optimal* in some sense, subject to a number of constraints and taking care of uncertainty in an explicit way. All these aspects are taken care of in a so-called *multistage recourse model*, which is a model for decision making under uncertainty belonging to the field of stochastic programming. Indeed, multistage recourse models have been applied successfully to a wide range of financial and other problems, see e.g. [18] and [13].

As detailed below, multistage recourse models comprise additional decisions which allow to react conditionally on new information, becoming available as the future unfolds. The corresponding additional or recourse variables come at certain unit costs, so that the risk associated with a current decision is modeled by assigning additional costs due to uncertainty. Alternatively, one may simply disallow decisions which are too risky (in some well-defined sense). We will use the latter approach to explicitly model *short-term risk* within a multistage recourse setting.

For the pension fund problem that we study in this paper, long-term solvency goals go together with short-term constraints on the *funding ratio*, defined as the ratio of assets over (discounted) future liabilities. Due to the uncertainty involved, such a constraint has to be stated in probabilistic terms. For example, it could be formulated as: The probability that the funding ratio one year from now falls below 105% should be at most 5%. In stochastic programming terminology, such restrictions on the feasible decisions are called probabilistic or *chance constraints*. They are closely related to the well-known *Value-at-Risk* concept used in financial applications.

The inclusion of chance constraints in multistage recourse models for pension fund ALM problems was pioneered by Dert [2]. In this paper we focus on an alternative formulation of short-term risk constraints in ALM models, known as *integrated chance constraints* (ICC). As detailed below, our motivation to advocate the use of ICCs comes both from modeling as well as computational considerations.

The remainder of this paper is organized as follows. First we outline the environment in which the problem is set. In Section 2 we then formalize our ALM decision problem for pension funds, arriving at a multistage recourse model. In Section 3 we motivate and describe in some detail the role and implementation of integrated chance constraints in this model. Moreover, we point out the relation between ICC and the more familiar *conditional surplus-at-risk* concept. Finally, in Section 4 we outline some of the other modeling features incorporated in our ALM model, and describe a heuristic for approximately solving instances of this very hard problem.

1.1 Dutch pension funds and regulation

Before entering upon these detailed issues, we first present some general background information on Dutch pension funds, since this is the setting of our ALM application. In The Netherlands, as in several other countries, old-age pensions consist of a state allowance complemented by payments out of pension savings. These savings are accumulated during each worker's active career by paying contributions (a fraction of the wages) to a pension fund, both by the employee and the employer. A pension fund may be related to or owned by a single company, a branch of industry, or a specific group of professionals.

Currently, there are about 830 pension funds in The Netherlands. Their total asset value is of the same order of magnitude as the Dutch GDP, see Table 1.1. While these data suggest that Dutch pension savings are at a relatively high level, the more recent absolute numbers of Table 1.2 indicate that there is ample reason for concern: following a period of very rapid growth, total asset value has been declining since 2000. On the other hand, liabilities are forecasted to grow steadily for several more years, mainly due to demographic developments. Indeed, although no recent official statistics are available, current estimates indicate that the funding ratio of 25% of the Dutch pension funds is too low, and that the accumulated funding shortage of this group of funds amounts to 23 billion Euros.

	Total assets/GDP (in %)
France	6.87
Germany	14.51
Italy	2.14
Netherlands	113.02
Spain	3.97
UK	79.06
US	104.95

Table 1.1: Total asset value of pension funds as a percentage of GDP (in 1997, source: OECD).

Thus, the situation has changed dramatically over the last few years. The Dutch regulating authority for pension funds (PVK, see `http://www.pvk.nl`) has reacted by adapting the rules by which pension funds have to operate. Although not all technical details are clear at this time, we anticipate that the PVK rules will be some implementation of the following three conceptual criteria:

(i) *Short term:* With high reliability, the funding ratio should be at least at some level specified by the PVK.

(ii) *Mid term:* Seen over a number of years, the funding ratio may fall short occasionally, but if this happens too often or if the shortage is too large (as defined by the PVK), some remedial action is required.

(iii) *Long term:* The solvency of the pension fund should be sufficiently high, from a going-concern and/or liquidation perspective.

Next we will see that all three criteria are covered in the multistage recourse ALM model that is outlined below. In particular, we will focus on the representation of the short-term criterion by means of integrated chance constraints.

2. A multistage recourse ALM model for pension funds

As explained in the introduction, our strategic ALM problem is a dynamic decision problem under uncertainty. We are asked to come up with decisions such as the contribution rate and the portfolio mix, and possibly a remedial action in case the funding ratio is insufficient. These decisions need to be taken right now, in the face of uncer-

Year	Total asset value
1997	333
1998	387
1999	451
2000	464
2001	456

Table 1.2: Total asset value ($\times 10^9$ €) of Dutch pension funds (source: PVK).

tainty about investment yields and other problem parameters. Obviously, to come up with a meaningful solution, we need to structure the problem and make some assumptions. The reformulation that we choose is known as a multistage recourse problem. Below we first review the underlying conceptual ideas and assumptions in the setting of our application, and then present some particular details. See the textbooks [1, 5, 11] and the handbook [13] for general discussions on formulation, properties, and solution approaches for stochastic programming models. Further information may be found on the Stochastic Programming Community Home Page at `http://stoprog.org` or in the bibliography [16].

2.1 Multistage recourse models

To capture the dynamics of the problem, we model the ALM process over a number of years; since we aim for strategic decisions, it suffices to allow for one set of decisions for each year. We *discretize time* accordingly, so that the model has a (finite) number of one-year time periods.

The next assumption, common to all stochastic programming models, is that the uncertain parameters can be modeled as *random variables* with known distribution.

At the start of each time period decisions can be made (corresponding to yearly corrections), and during each one-year period a realization of the corresponding random parameters becomes known (e.g. the yield of stocks during that year). That is, the concept underlying our model is a *sequence of decisions and observations*. At time t, $t = 0, 1, \ldots, T$ (with $t = 0$ denoting 'now' and $t = T$ at the planning horizon), decisions x_t are taken with full knowledge of the past $[0, t)$ but with only probabilistic information about the future $(t, T]$. Denoting ω_t, $t = 1, \ldots, T$, the vector of random parameters whose realization is revealed during the t-th year (the time interval

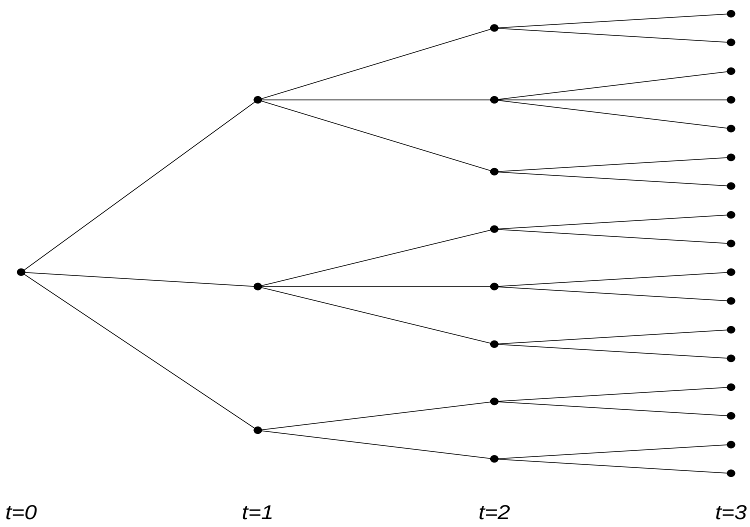

Figure 2.1: A scenario tree with 17 scenarios and 29 nodes.

$(t-1, t)$), we have a discrete-time random process $(\omega_1, \dots, \omega_T)$ modeling the uncertainty. We assume that there are only *finitely many realizations* of this process, denoted $(\omega_1^s, \dots, \omega_T^s)$ with corresponding known probabilities p^s for $s \in \mathcal{S} := \{1, \dots, S\}$.

Such a realization of the random process is called a *scenario*, and can be interpreted as a description of a possible future, starting just after $t = 0$. Now assume for a moment that we can observe the 'state of the world' at time t, $0 < t < T$. Then there is a unique history of realizations of $(\omega_1, \omega_2, \dots, \omega_{t-1})$ leading to that state, but the future as seen from time t may unfold in several ways. In other words, there are several distinct scenarios which share a common history up to time t. This relation between scenarios is easy to see when they are presented as a *scenario tree*, as depicted in Figure 2.1. Each scenario is a path in this directed graph from the root node at $t = 0$ to an end node at $t = T$; conventionally, the scenarios are numbered top-down by their end node. The arcs in the tree denote realizations in one time period. For example, in the tree of Figure 2.1 there are 3 distinct arcs starting from the node at time $t = 1$ in scenario 1. They represent the 3 possible realizations of the random parameters during the period $(1, 2)$ given that this is the observed state at time $t = 1$; there are 7 partial scenarios describing the remaining future.

A *multistage recourse model* is an optimization problem defined on such a scenario tree, as follows. At every node (t, s) of the tree, decisions are to be made which are optimal given the history up to then and under uncertainty about the remaining future, represented by the subtree rooted at (t, s). Optimality is defined in terms of current costs plus expected future costs, which are computed with respect to the appropriate conditional distributions. In particular, the decisions at the unique node at time $t = 0$ (now) will be optimal in this sense; thus, the entire tree can be seen as a sophisticated way to model the expected future costs associated with any current decision. It is important to observe that only the decisions for $t = 0$ are to be implemented; using a rolling horizon approach, future decisions will be reconsidered at time $t = 1$.

Below we will conveniently use x_t^s to indicate the decisions to be taken in node (t, s). However, it follows from the preceding discussion that this notation is actually not correct, since there are usually several scenarios going through the node (t, s) (unless $t = T$). In the tree representation of the problem used here, this ambiguity may be resolved by numbering the nodes uniquely, but the resulting notation will not be as easy to interpret. In the alternative scenario representation of the problem, so-called nonanticipativity constraints are included to enforce the required structure.

At every node (t, s), the set of feasible decisions x_t^s is defined through constraints, some of which reflect the consequences of past decisions and observations. Other constraints may be used to model particular relations between the variables x_t^s. We will assume that *all constraints are linear*, as is common practice in multistage recourse modeling. Accordingly, the current-costs part of the objective function is a linear function of the decision variables x_t^s. Given the well-earned reputation of linear programming, this still allows for ample modeling flexibility.

2.2 An ALM model for pension funds

In our multistage ALM model, each nodal subproblem (t, s) describes the setting in which strategic decisions have to be made for year $t + 1$ (i.e., the period $(t, t + 1)$), given the observed history and the uncertain future as perceived in state (t, s). Among others, these decisions involve

- the contribution rate,
- the amounts to be invested in various asset classes (e.g., stocks, bonds, real estate, cash),
- the indexation level of future pension payments, and
- a remedial contribution (if any) by the sponsor of the fund.

Note that we assume the existence of a sponsor. Later on, it will become clear why this assumption is made.

The subproblem setting is given by the cost function, consisting of current plus expected discounted future costs, and a number of constraints describing

- actuarial principles,
- laws,
- company policies, and
- criteria stipulated by the regulator PVK.

Of course, the first three types of constraints are common to all ALM models for pension funds. Below we focus on our implementation of Dutch regulatory rules, which were already presented in Section 1.1.

The long-term criterion, stated in terms of solvency of the fund, is taken care of by the very nature of our multistage recourse model. Indeed, by choosing a suitably long planning horizon, any reasonable planning period can be represented explicitly in the model. However, for computational and modeling reasons, the current version of our model is restricted to four periods of one year each, corresponding to five decision moments (now, and after each year up to the planning horizon). The more distant future is modeled by including suitable solvency target levels at the planning horizon.

We model the mid-term restrictions on the funding ratio by keeping track of years in which underfunding occurs; based on this information, a remedial contribution by the sponsor may be called for. More details will be presented in Section 4.

Our modeling of the short-term criterion, formulated as a probabilistic restriction on next years funding ratio, will be set out in detail in the next section.

A complete description of a preliminary version of our model, including discussion on modeling issues, can be found in [4]. An updated version of the model and numerical results for a set of small test problems are presented in [3]. Moreover, a complete description of the current model, including numerical results for a semi-realistic problem instance, will be published in the Ph.D. thesis of S.J. Drijver (University of Groningen, forthcoming).

3. Integrated chance constraints

The short-term criterion proposed by the PVK reads that next years funding ratio F_{t+1} should be on or above a given level α, say $\alpha = 105\%$, with *high reliability*. The formulation of this criterion clearly indicates its stochastic nature, which should be reflected in the way it is modeled. Starting from traditional chance constraints, we will arrive at our implementation of an adapted short-term criterion by means of integrated chance constraints.

3.1 Conceptual motivation

In our ALM model, a direct translation of the short-term criterion would be the *chance constraint*

$$\Pr\{F_{t+1} \geq \alpha | (t, s)\} \geq \gamma_t,$$

where γ_t is the required reliability at time t (e.g. $\gamma_t = 0.95$), and the notation indicates that probability is measured conditional on (t, s) being the current node. Recalling that the funding ratio is defined as the ratio of the assets A_{t+1} over liabilities L_{t+1}, an equivalent formulation is

$$\Pr\{A_{t+1} - \alpha L_{t+1} \geq 0 | (t, s)\} \geq \gamma_t. \tag{1}$$

Note that the nature of such a chance constraint is *qualitative*, in the sense that it measures the probability of a shortfall of the funding ratio, but the magnitude of the shortage is not taken into account. In other applications this may be justified or even preferable, but in our ALM model the size of the funding shortage is obviously relevant.

Of course, this criticism actually applies to the underlying short-term criterion suggested the PVK, which should take the magnitude of funding shortages into account. Thus, we propose to replace the current criterion by a *quantitative* one-year risk measure. In our ALM model, this role is played by *integrated chance constraints*. To introduce this concept, we return to the chance constraint, and will see that it comes up in a natural way when we look at computational issues.

3.2 Computational motivation and definition

In the chance constraint (1), both A_{t+1} and L_{t+1} are random quantities because they depend on underlying random parameters such as asset yields. Moreover, they depend linearly on the current decisions x_t^s which involve asset allocation and indexation. To stress these relations and in order to simplify the notation, for the time being we will use the generic representation of an (individual) chance constraint

$$\Pr\{Bx - d \geq 0\} \geq \gamma,$$

where x is an n-vector of decision variables and the n-vector B and the scalar d are both random parameters. As in our ALM model, we assume that (B, d) follow a finite discrete distribution with realizations (B^s, d^s) and corresponding probabilities p^s, $s \in \mathcal{S} := \{1, \ldots, S\}$.

It was noted above that all other constraints in our ALM model are linear, and also the chance constraint can be represented by linear constraints, as follows.

$$
\begin{aligned}
& B^s x + \delta^s M \geq d^s, \quad s \in \mathcal{S} \\
& \sum_{s \in \mathcal{S}} p^s \delta^s \leq 1 - \gamma \\
& \delta^s \in \{0, 1\}, \quad s \in \mathcal{S},
\end{aligned}
$$

where M is a sufficiently large number. Note however, that this formulation necessarily uses *binary variables* δ^s, $s \in \mathcal{S}$, to indicate realizations (B^s, d^s) which are unfavorable for x, e.g., which would result in underfunding in the ALM model. The probability weighted average of these binary variables then equals the risk of underfunding associated with the decision x, which should be at most $1 - \gamma$. Because of these binary variables, the requested inclusion of a chance constraint at every node (t, s) (except at the end nodes) of our multistage recourse model would have severe consequences for the computational tractability of the model.

For problems involving binary (or general integer) decision variables, a natural approach is to relax the integrality restrictions and solve the resulting relaxation. In our case, such a relaxation transforms the mixed-integer linear representation of the chance constraint into a system of linear constraints in continuous variables, which is equivalent to

$$
\begin{aligned}
& B^s x + y^s \geq d^s, \quad s \in \mathcal{S} \\
& \sum_{s \in \mathcal{S}} p^s y^s \leq \beta \\
& y^s \geq 0, \quad s \in \mathcal{S},
\end{aligned}
\tag{2}
$$

where the parameter β is non-negative. By the first set of inequalities, each of the non-negative variables y^s is not less than the shortfall $d^s - B^s x$ (if any). The next inequality therefore puts an upper bound β on the *expected shortfall*. That is, the system (2) is equivalent to

$$
\sum_{s \in \mathcal{S}} p^s \left(B^s x - d^s\right)^- \leq \beta,
$$

where $(a)^- := \max\{-a, 0\}$ is the negative part of $a \in \mathbb{R}$, or

$$\mathbb{E}\left[(Bx - d)^-\right] \leq \beta \tag{3}$$

with $\mathbb{E}$ denoting expectation with respect to the distribution of (B, d). Such constraints, bounding an expected shortfall, were named *integrated chance constraints* by Klein Haneveld [6], since it can be shown that

$$\mathbb{E}\left[(Bx - d)^-\right] = \int_{-\infty}^{0} \Pr\{Bx - d < u\} du. \tag{4}$$

The integrand in (4) is the complement of the probability $\Pr\{Bx - d \geq u\}$; it appears in the equivalent risk version

$$\Pr\{Bx - d < u\} \leq 1 - \gamma(u)$$

of the underlying chance constraint formulated for the target level u.

Loosely speaking, the identity (4) shows that an integrated chance constraint corresponds to some aggregation of the infinitely many chance constraints which – in theory – could be defined for all possible target levels $u \leq 0$. By indicating, through the corresponding risk parameters $1 - \gamma(u)$, that larger shortages are even less acceptable than smaller ones, this indeed results in a quantitative risk measure defined in terms of traditional, qualitative chance constraints.

In [8] it is shown that integrated chance constraints are closely related to constraints on the *conditional surplus-at-risk* (CSaR), which is a variant of conditional value-at-risk, see e.g. [15]. Essentially, the difference is that in an ICC constraint the shortage is measured with respect to some a priori chosen threshold parameter, whereas in a CSaR constraint the threshold is equal to the surplus-at-risk, which is itself an outcome of the optimization process.

3.3 Conclusions and implementation

We conclude that integrated chance constraints provide a suitable way to model short-term risk in our ALM model, both from conceptual as well as computational point of view. Returning to the specific notation of our ALM model, we thus include an integrated chance constraint

$$\mathbb{E}\left[(A_{t+1} - \alpha L_{t+1})^- \,|(t, s)\right] \leq \beta_t \tag{5}$$

in every subproblem (t, s), $t < T$, of our multistage recourse model. They reflect our alternative short-term criterion, stating that *next years funding ratio should be such that* *the expected funding shortfall is at most* β_t, given that the current state is (t, s).

The parameters β_t, $t = 0, \ldots, T-1$, giving the maximal acceptable expected funding shortage, of course need to be specified numerically. It should be noted that in general it is harder to come up with these values than it would be for the reliability parameters α_t, as required for traditional chance constraints. The latter parameters are scale free, and correspond to a risk notion which is more familiar to e.g. managers of pension funds. In our ALM model, we compute the parameters β_t as fractions of the expected discounted liabilities at $t = 0$, with the fraction depending on the duration of the liabilities.

With S_t^s denoting the number of possible realizations in year $t+1$, it follows from (2) that inclusion of the integrated chance constraint (5) in subproblem (t, s) comes at the price of S_t^s additional continuous variables and $S_t^s + 1$ additional linear constraints. In our ALM model, S_t^s is in the order of 5 to 10, so that this extension of the model provides no computational hardship.

In case the number of realizations is substantially larger, say 1000 or more, the linear programming (LP) representation (2) becomes inefficient. In [8] we showed that the induced feasible set $C(\beta)$, corresponding to an integrated chance constraint (3) with an underlying finite discrete distribution on S points, is given by

$$C(\beta) = \bigcap_{K \subset \mathcal{S}} \left\{ x \in \mathbb{R}^n : \sum_{k \in K} p^k \left(d^k - B^k x \right) \leq \beta \right\}. \tag{6}$$

Since there are $2^S - 1$ non-empty subsets of $\mathcal{S} = \{1, \ldots, S\}$, it follows that $C(\beta)$ is a polyhedral set defined by as many linear constraints. For any non-trivial number of realizations S, it is obviously not sensible (if at all possible) to explicitly include all of them in the model. However, the representation (6) underlies a very efficient cutting plane algorithm for solving LP problems with (variants of) integrated chance constraints. For a small example problem with 1000 realizations, an optimal solution is found after generating only 9 out of the approximately 10^{300} constraints defining the set $C(\beta)$. Further numerical evidence shows that the cutting plane algorithm is much faster than the straightforward LP approach on larger problem instances.

4. Other characteristics and a heuristic for our ALM model

In this final section, we briefly describe some other characteristics of our ALM model. In particular, we discuss how we model the mid-term criterion proposed by the PVK.

Finally, we outline our heuristic solution approach for the resulting multistage recourse ALM model, which – as we will see – involves binary decision variables after all.

Foremost, the introduction of binary variables in the model is justified by the fact that they are necessary to include several realistic features in our multistage recourse ALM model. In addition, we have a long-standing interest in (computational) properties of recourse models with (mixed-)integer variables, see e.g. [10, 7, 17]. In fact, our interest in this ALM application was raised initially by the observation that integer variables appear naturally in these models.

4.1 Modeling remedial action in case of underfunding

For the mid-term range, the PVK criterion (or at least, our interpretation of it) allows that the funding ratio occasionally drops below the required level α, but this should not happen too often nor should the shortage be too large. Given the long-term character of the liabilities, this appears to be very reasonable: there is no compelling reason to react immediately on a possibly temporary drop in e.g. stock prices. Indeed, such an immediate reaction could very well turn out to be harmful in the longer run. On the other hand, if the funding ratio is too far below the aspired level α, then immediate action might be necessary to prevent a further decline.

In the current version of our ALM model, 'not too often' is understood as 'not in two consecutive years'. However, with only minor modifications, the approach outlined below also applies to the more general interpretation 'at most in n out of m consecutive years'. To simplify the exposition, we do not account here for immediate action in case of a very low funding ratio. It is not difficult to see how this refinement can be modeled.

Essentially, our approach is based on keeping track of years in which underfunding occurs. This calls for the use of binary *indicator variables* in the model. In turn, these indicator variables are used to determine when a remedial action is necessary in order to restore the funding ratio to the required level α. In our model, remedial contributions by the sponsor of the pension fund are used to this end.

With A_t^s and L_t^s denoting assets and liabilities in node (t, s) as before, the constraint

$$M\delta_t^s \geq \alpha L_t^s - A_t^s,$$

where M is again a sufficiently large number, forces the indicator variable δ_t^s to take on the value 1 in case of a funding shortage. If this happens in two consecutive years in the same scenario s, then the constraint

$$\pi_t^s \geq \delta_{t-1}^s + \delta_t^s - 1$$

forces the indicator variable π_t^s to become 1, triggering a remedial contribution R_t^s, equal to the observed current shortage $\alpha L_t^s - A_t^s$, which is modeled by the constraint

$$R_t^s \geq \alpha L_t^s - A_t^s - M(1 - \pi_t^s).$$

In addition, the model contains constraints which ensure that a remedial contribution is only allowed in case of underfunding. Because both funding shortages and remedial contributions are undesirable events, corresponding fixed penalty costs are included in the objective function. On top of that, proportional costs for remedial contributions are assigned.

On the opposite side, if the pension fund has a structural funding surplus, a restitution to the sponsor may be required. This is modeled analogously to the shortage case above. (Unfortunately, this issue is currently not as relevant as when we developed a first version of the model a few years ago.) Further realistic features of our ALM model include a detailed modeling of indexation of future pension payments (again using indicator variables), and the use of *soft constraints*, for example to model a preference for gradual changes in the contribution rate from one year to the next. For a detailed exposition of all features included in the model we refer to [4, 3] and the forthcoming Ph.D. thesis of Drijver.

4.2 Heuristic solution procedure

This realistic modeling of the ALM problem for Dutch pension funds comes at a high price: the resulting model is a multistage recourse problem with mixed-integer decision variables. Although some progress has been made in the last decade, see e.g. the survey papers [7, 12, 9, 14], such problems are extremely difficult to solve in general. Therefore, instead of aiming to find an optimal solution, we developed a special-purpose heuristic incorporating our insight in the ALM decision problem. The resulting feasible solutions are hoped to be reasonably good or even near-optimal.

The basic steps of this heuristic are:

(i) Drop the integrality restrictions on all binary variables, and solve the resulting multistage linear programming (MSLP) problem.

(ii) Round all relaxed binary variables, according to their definitions and interpretations. This rounding procedure is rather involved because of the many relations between variables and the possible consequences for future nodes in the scenario tree.
Re-solve the MSLP problem, keeping the values of the binary variables fixed.

(iii) In a greedy way, identify a subproblem (i.e., a node in the tree) with positive fixed costs associated with the binary variables, and try to reduce the total costs by changing one or more binary variables from 1 to 0. For example, if a remedial contribution appears in node (t, s), check whether it is beneficial to remove it and instead increase the contribution rate in one or more nodes (τ, s), $0 \leq \tau < t$, if at all possible. Also this step is complicated, because all consequences for affected parts of the tree need to be accounted for (i.e., in all nodes belonging to the subtree rooted at (τ, s)).

If such a cost reduction is found, fix the updated binary variables and solve the MSLP problem again to find optimal continuous variables corresponding to this setting of the binaries. This optimization step is useful because some of the continuous variables (in particular, the asset portfolio mix) were kept fixed during the modification of the binary variables.

Step (iii) is repeated as long as an improvement in the objective value of the MSLP problem is found.

To solve the MSLP problems, we use the OSL Stochastic Extensions library provided by IBM (King et al.), see `http://www-3.ibm.com/software/data/bi/osl/features/stex.html`. The heuristic itself is programmed in C++.

Preliminary computations indicate that by far the most computing time of our heuristic is spent in Step (i), where the MSLP problem is solved for the first time. Both the modification of binary variables as well as re-solving the MSLP with updated (fixed) binaries (using a hot start) are relatively fast. Thus, repeated execution of Step (iii) of the heuristic appears to be feasible from computational point of view.

No computational results on (semi-)realistic data are available at this time. However, such data have been made available to us by a major Dutch pension fund. Outcomes of our ALM model and performance of the heuristic for this data set will be reported in the forthcoming Ph.D. thesis of Drijver, and in other publications.

5. Summary and concluding remarks

We motivated and described the role played by integrated chance constraints in our ALM model for Dutch pension funds. To set the stage, we outlined the practical setting as well as our modeling approach for this dynamic decision problem under uncertainty.

Integrated chance constraints are appropriate for modeling single-period risk constraints, in particular if a quantitative risk measure preferable, as is the case here. Moreover, they are computationally attractive in the given multistage recourse setting, since they can be formulated in terms of a limited number of linear constraints.

We expect that our model will prove to be a useful tool in strategic ALM studies for pension funds. Even though we took great care to model important aspects in a realistic way, the final judgement on our model will have to come from the analysis of numerical results for (semi-)realistic problems.

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